

QUIZ 3: LESSON 2
AUGUST 27, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [10 pts] A certain plant grows at a rate of

$$H'(t) = t\sqrt{t^2 + 1} \text{ inches/day}$$

t days after it is planted. By how many inches will the plant change on the **second** day? Round your answer to the nearest hundredth.

Solution: t is measured in days after the plant is planted, which means

$$\underbrace{0 \leq t < 1}_{\text{Day 1}} \text{ and } \underbrace{1 \leq t < 2}_{\text{Day 2}}.$$

Thus, the change of the height of the plant on the second day is given by

$$\int_1^2 t\sqrt{t^2 + 1} dt.$$

This integral is a u -sub problem. Let $u = t^2 + 1$, then

$$du = 2t dt \Rightarrow \frac{du}{2t} = dt.$$

Since we have bounds, we will need to convert:

$$u(1) = 1^2 + 1 = 2 \quad \text{and} \quad u(2) = 2^2 + 1 = 5.$$

Now, we may write

$$\begin{aligned} \int_1^2 t\sqrt{t^2 + 1} dt &= \int_{u(1)}^{u(2)} t\sqrt{u} \left(\frac{du}{2t} \right) \\ &= \int_2^5 \frac{1}{2} \sqrt{u} du \\ &= \int_2^5 \frac{1}{2} u^{1/2} du \\ &= \frac{1}{2} \left(\frac{1}{1/2 + 1} \right) u^{1/2+1} \Big|_2^5 \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \left(\frac{1}{3/2} \right) u^{3/2} \Big|_2^5 \\ &= \frac{1}{2} \left(\frac{2}{3} \right) u^{3/2} \Big|_2^5 \\ &= \frac{1}{3} u^{3/2} \Big|_2^5 \\ &= \frac{1}{3} [(5)^{3/2} - (2)^{3/2}] \\ &\approx \boxed{2.78} \end{aligned}$$